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MTH 111, Final Exam

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$$SCORE = \frac{61}{67}$$

QUESTION 1. (15 points). Find y' and do not simplify

(i) $y = (3x^2 + 6x + 2)e^{(x+1)}$

$y' = (6x+6)\cdot e^{(x+1)} + e^{(x+1)} \cdot 1\,(3x^2+6x+2)$

(ii) $y = (2x^5 + 6x + 3)^4$

$y' = 4(2x^5+6x+3)^3 \cdot (10x^4+6)$

(iii) $y = ln\left(\frac{(3x+7)^7}{(2x+1)^3}\right)$ $y = 7\ln(3x+7) - 3\ln(2x+1)$

$y' = \frac{21}{3x+7} - \frac{6}{2x+1}$

(iv) $y = ln((3x+2)^3(7x+2)^6)$ $y = 3\ln(3x+2) + 6\ln(7x+2)$

$y' = \frac{9}{3x+2} + \frac{42}{7x+2}$

(v) Given $f(x) = k(5x+1)$ and $k'(11) = 4$. Find $f'(2)$

$f(x) = k(5x+1)\cdot(5)$ $f'(2) = k(11)\cdot 5$ $f'(2) = 20$

$f'(2) = k(5(2)+1)\cdot(5)$ $4 \cdot 5 = 20$

QUESTION 2. (5 points). Find the equation of the plane that passes through $Q_1 = (1,2,3), Q_2 = (-1,0,2),$ *and* $Q_3 = (4,3,2)$.

$U = \overrightarrow{Q_1Q_2} \to \langle -2, -2, -1 \rangle$

$w = \overrightarrow{Q_1Q_3} \to \langle 3, 1, -1 \rangle$

$$\begin{vmatrix} i & j & k \\ -2 & -2 & -1 \\ 3 & 1 & -1 \end{vmatrix}$$

$$\left\langle \begin{vmatrix} -2 & -1 \\ 1 & -1 \end{vmatrix}, -\begin{vmatrix} -2 & -1 \\ 3 & -1 \end{vmatrix}, \begin{vmatrix} -2 & -2 \\ 3 & 1 \end{vmatrix} \right\rangle$$

$(-2)(-1) - (-1)(1)$, $-(-2)(-1) - (-1)(3)$, $(-2)(1) - (-2)(3)$

$\langle 3, -5, 4 \rangle \cdot (x-1, x-2, x-3)$

$3(x-1) - 5(x-2) + 4(x-3)$

$3x - 3 - 5x + 10 + 4x - 12$

$\boxed{3x - 5x + 4x = 5}$

QUESTION 3. (7 points). The plane $P_1 : x + 2y + z = 4$ intersects the plane $P_2 : -x - y + 2z = 6$ in a line L. Find the parametric equations of L, and then find the symmetric equation of L.

$P_1 \quad \langle 1, 2, 1 \rangle$

$P_2 \quad \langle -1, -1, 2 \rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -1 & -1 & 2 \end{vmatrix} \quad \left\langle \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}, -\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ -1 & -1 \end{vmatrix} \right\rangle$$

$(2)(2)-(1)(-1), -(1)(2)-(1)(-1), (1)(-1)-(2)(-1)$

$x = 0$

1) $2y + z = 4$

2) $-y + 2z = 6$

$$\begin{array}{c|c} c & z \\ 4 & 1 \\ 6 & 2 \\ \hline y & z \\ 2 & 1 \\ -1 & 2 \end{array} \qquad \frac{(4)(2)-(1)(6)}{(2)(2)-(1)(-1)}$$

$\langle 5, -3, 1 \rangle$

$y = \frac{2}{5}$

$2\left(\frac{2}{5}\right) + z = 4$

$\frac{4}{5} + z = 4$

$z = 4 - \frac{4}{5}$

$z = \frac{16}{5}$

$\left(0, \frac{2}{5}, \frac{16}{5}\right)$

Parametric

$x = 5t$

$y = -3t + \frac{2}{5}$

$z = 1t + \frac{16}{5}$

symmetric

$$\frac{x}{5} = \frac{y - \frac{2}{5}}{-3} = \frac{z - \frac{16}{5}}{1}$$

QUESTION 4. (5 points). Let $f(x) = ln(5x - 9) + 3e^{(3x-6)} + 3x^2 - 6$. Find the equation of the tangent line to the curve of $f(x)$ when $x = 2$.

$f'(x) = \frac{5}{5x-9} + 3e^{(3x-6)} \cdot 3 + 6x$

$f'(2) = \frac{5}{5(2)-9} + 3e^{(3(2)-6)} \cdot 3 + 6(2)$

$= 26 \qquad y = 26x + b$

$f(x) = \ln(5(2)-9) + 3e^{(3(2)-6)} + 3(2)^2 - 6$

$= 9$

$9 = 26x + b$

$9 = 26(2) + b$

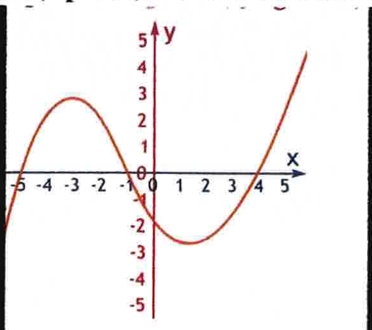
$9 - 52 = b$

$-43 = b$

$\boxed{y = 26x - 43}$

QUESTION 5. (8 points). Consider the curve of $f'(x)$ as below

[redacted]



[redacted]

sign of $f'(x)$

$- - - \times + + + \times - - - \times + + +$

$-5 \qquad -1 \qquad 4$

(i) Stare at the curve , for what values of x does $f(x)$ increase?

$(-5,-1) \cup (4,\infty)$ ~~$(-\infty,-5)$ $(-1,4)$~~

-6

(ii) Stare at the curve, for what values of x does $f(x)$ decrease?

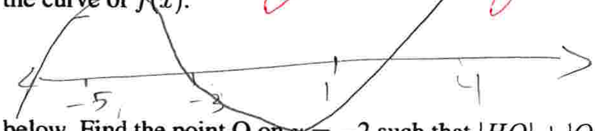
~~$(-1,1)$~~ $(-\infty,-5) \cup (-1,4)$

stare

(iii) Stare at the curve, for what values of x does $f(x)$ have local max, local min?

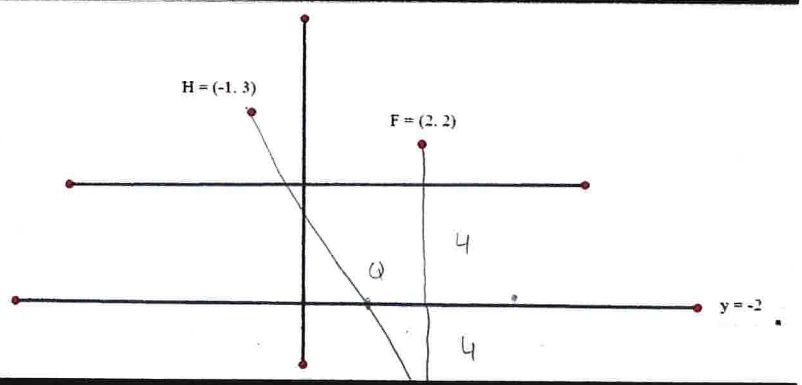
max at ~~$x=4$~~ $x=-1$ local min at -5 and 4

(iv) Stare at the curve, roughly, sketch the curve of $f(x)$.



QUESTION 6. (6 points). Stare at the below. Find the point Q on $y=-2$ such that $|HQ|+|QF|$ is minimum.

[redacted]



[redacted]

$=(2,-6)$

$Q = \left(\frac{2}{3}, -2\right)$

$\frac{\Delta y}{\Delta x} = \frac{-9}{3} = -3$

$y = -3x + b$

$3 = -3(-1) + b$

$3 - 3 = b$

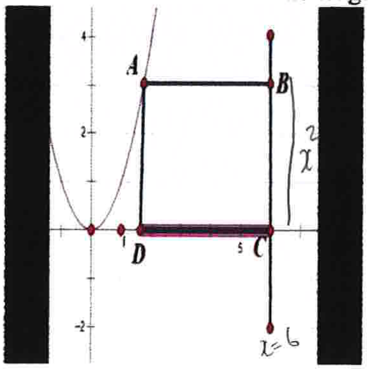
$0 = b$

$-2 = -3x$

$\frac{-2}{-3} = \frac{-3x}{-3}$

$\frac{2}{3} = x$

QUESTION 7. (6 points). Stare at the picture below. Given A is on the curve $y = x^2$, the points B, C are on the line $x = 6$ and D is on the x-axis. Find the length AD and the width DC so that the area of the rectangle $ABCD$ is maximum.



$y = x^2$
$x = 6$

$A = x^2(6-x) = 6x^2 - x^3$

$A' = 12x - 3x^2$

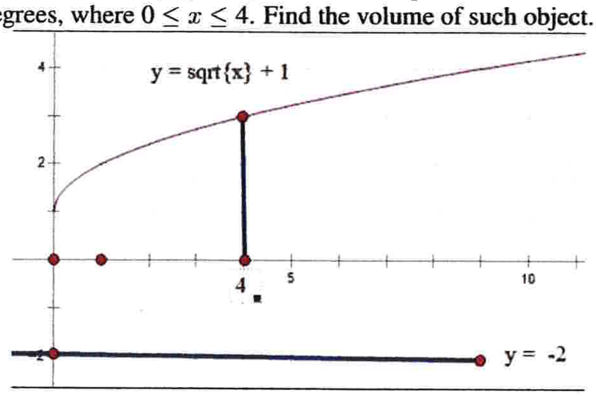
$12x - 3x^2 = 0$
$x = 4$
$\cancel{x = 0}$

$A''(x) < 0$
$= 12 - 6(4)$
$= -12$

$L = 4^2 = 16$
$w = 6 - x$
$= 6 - 4 = 2$

Area $= 16 \times 2 = 32$

QUESTION 8. (5 points). Stare at the picture below. We rotate the curve $y = \sqrt{x} + 1$ around $y = -2$, 360 degrees, where $0 \le x \le 4$. Find the volume of such object.



$\int_0^4 \pi (x^{\frac{1}{2}} + 1 + 2)^2 \, dx$

$\int_0^4 \pi (x^{\frac{1}{2}} + 3)^2 \, dx$

$\pi \int_0^4 (x + 6x^{\frac{1}{2}} + 9)$

$\frac{1}{2}x^2 + 4x^{\frac{3}{2}} + 9x \Big|_{x=0}^{x=4} \quad \frac{1}{2}(4)^2 + 4(4)^{\frac{3}{2}} + 9(4)$

$\boxed{= 76\pi}$

QUESTION 9. (5 points). Given $f(x) = \frac{3}{2}\sqrt{x} + 2x + 4$ is above the x-axis when $0 \le x \le 4$. Find the area of the region bounded by $f(x)$, x-axis, and $0 \le x \le 4$.

$A = \int_0^4 \frac{3}{2} x^{\frac{1}{2}} + 2x + 4 \, dx$

$\int_0^4 x^{\frac{3}{2}} + x^2 + 4x \Big|_{x=0}^{x=4} \quad (4)^{\frac{3}{2}} + 4^2 + 4(4) - 0$

$\boxed{= 40}$

QUESTION 10. (5 points). (1) Find $\int e^{(4x+2)} + \frac{10}{x} + 4 \, dx$.

$\frac{1}{4} e^{(4x+2)} + 10 \ln|x| + 4x + C$

(2) Find $\int \frac{4}{x^3} + \sqrt{x} + 2x \, dx$

$\int 4x^{-3} + x^{\frac{1}{2}} + 2x$

$\frac{4}{-2} x^{-2} + \frac{2}{3} x^{\frac{3}{2}} + \frac{2}{2} x^2 + C \rightarrow -2x^{-2} + \frac{2}{3} x^{\frac{3}{2}} + x^2 + C$